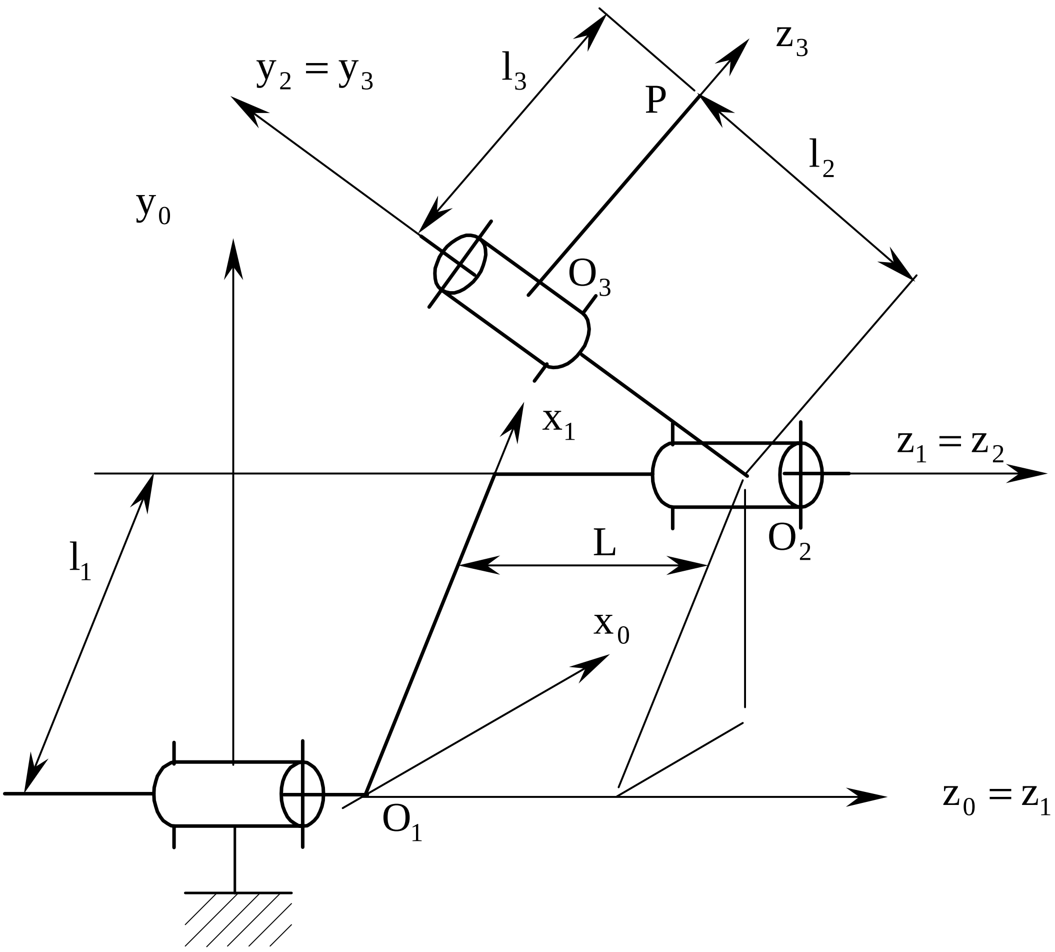
TD2 : LE MANEGE MAGIC ARMS
Eléments de correction

Le paramétrage utilisé est donné sur le schéma cinématique suivant :



Le Manège "Magic Arms"

Question 1 : On obtient :

$\overrightarrow{O_1O_2} = l_1\vec{x}_1 + L\vec{z}_1$ $\overrightarrow{O_2O_3} = l_2\vec{y}_2$ $\overrightarrow{O_3P} = l_3\vec{z}_3$

Question 2 :

Tracer les figures de calcul et exprimer les vecteurs taux de rotation $\vec{\Omega}(S_i / S_j)$ en fonction des dérivées des variables articulaires $\dot{\theta}_{ij}$.

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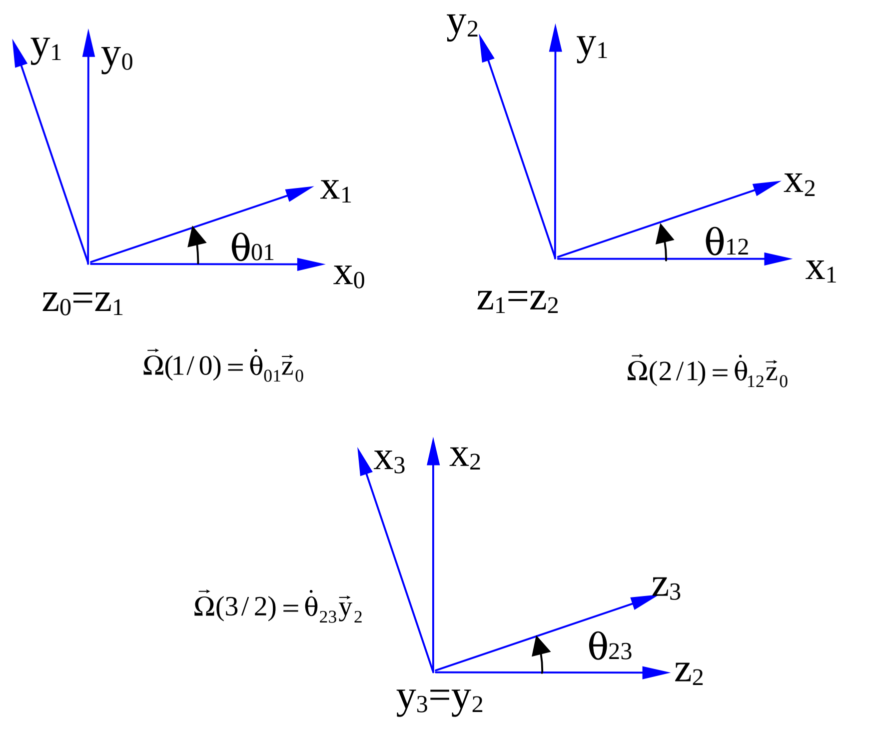

Figures de calcul

Question 3 : On trouve bien sûr :

$$\boxed{\begin{aligned}\vec{\Omega}(S_1/R_0) &= \dot{\theta}_{01}\vec{z}_0\\ \vec{\Omega}(S_2/R_0) &= (\dot{\theta}_{12}-\dot{\theta}_{01})\vec{z}_0\\ \vec{\Omega}(S_3/R_0) &= \dot{\theta}_{23}\vec{y}_2\end{aligned}}$$

Question 4 :

En cinématique du point :

$$\begin{aligned}\vec{V}(P,S_3/R_0) &= \vec{V}(P/R_0)-\vec{V}(P/R_3)=\vec{V}(P/R_0) \text{ car } P\in S_3\\ &= \left(\frac{d\overrightarrow{O_1P}}{dt}\right)_0 = l_1\left(\frac{d\vec{x}_1}{dt}\right)_0 + l_2\left(\frac{d\vec{y}_2}{dt}\right)_0 + l_3\left(\frac{d\vec{z}_3}{dt}\right)\\ &= l_1\dot{\theta}_{01}\vec{y}_1 + l_2(\dot{\theta}_{01}+\dot{\theta}_{12})\vec{z}_1\wedge\vec{y}_2 + l_3\left[(\dot{\theta}_{01}+\dot{\theta}_{12})\vec{z}_1+\dot{\theta}_{23}\vec{y}_2\right]\wedge\vec{z}_3\\ \vec{V}(P,S_3/R_0) &= l_1\dot{\theta}_{01}\vec{y}_1 - l_2(\dot{\theta}_{01}+\dot{\theta}_{12})\vec{x}_2 + l_3\left[(\dot{\theta}_{01}+\dot{\theta}_{12})\sin\theta_{23}\vec{y}_2+\dot{\theta}_{23}\vec{x}_3\right]\end{aligned}$$

$$\boxed{\vec{V}(P,S_3/R_0)=l_1\dot{\theta}_{01}\vec{y}_1 - l_2(\dot{\theta}_{01}+\dot{\theta}_{12})\vec{x}_2 + l_3\left[(\dot{\theta}_{01}+\dot{\theta}_{12})\sin\theta_{23}\vec{y}_2+\dot{\theta}_{23}\vec{x}_3\right]}$$

En cinématique du solide :

$$\begin{aligned}\vec{V}(P,S_3/R_0) &= \vec{V}(P,S_3/S_2)+\vec{V}(P,S_2/S_1)+\vec{V}(P,S_1/S_0)\\ &= \vec{V}(O_3,S_3/S_2)+\overrightarrow{PO_3}\wedge\vec{\Omega}(S_3/S_2)+\vec{V}(O_2,S_2/S_1)+\overrightarrow{PO_2}\wedge\vec{\Omega}(S_2/S_1)+\\ &\quad \vec{V}(O_1,S_1/S_0)+\overrightarrow{PO_1}\wedge\vec{\Omega}(S_1/S_0)\\ \vec{V}(P,S_3/R_0) &= -l_3\vec{z}_3\wedge\dot{\theta}_{23}\vec{y}_2+(-l_3\vec{z}_3-l_2\vec{y}_2)\wedge\dot{\theta}_{12}\vec{z}_1+(-l_3\vec{z}_3-l_2\vec{y}_2-L\vec{z}_1-l_1\vec{x}_1)\wedge\dot{\theta}_{01}\vec{z}_1\end{aligned}$$

soit finalement le même résultat que précédemment :

$$\boxed{\vec{V}(P,S_3/R_0)=l_3\dot\theta_{23}\vec{x}_3+l_3\dot\theta_{12}\sin\theta_{23}\vec{y}_2-l_2\dot\theta_{12}\vec{x}_2+l_3\dot\theta_{01}\sin\theta_{23}\vec{y}_2-l_2\dot\theta_{01}\vec{x}_2+l_1\dot\theta_{01}\vec{y}_1}$$

Question 5 :

$$\begin{aligned}
\vec{\Gamma}(P,S_3/R_0) &= \vec{\Gamma}(P/R_0)+\vec{\Gamma}(P/R_3)+2\vec{\Omega}(S_3/R_0)\wedge\vec{V}(P/R_3)=\vec{\Gamma}(P/R_0)\\
&= l_3\ddot\theta_{23}\vec{x}_3+l_3\dot\theta_{23}\left[\frac{d\vec{x}_3}{dt}\right]_0+l_3(\ddot\theta_{12}+\ddot\theta_{01})\sin\theta_{23}\vec{y}_2+l_3(\dot\theta_{12}+\dot\theta_{01})\dot\theta_{23}\cos\theta_{23}\vec{y}_2\\
&+ l_3(\dot\theta_{12}+\dot\theta_{01})\sin\theta_{23}\left[\frac{d\vec{y}_2}{dt}\right]_0-l_2(\ddot\theta_{12}+\ddot\theta_{01})\vec{x}_2-l_2(\dot\theta_{12}+\dot\theta_{01})\left[\frac{d\vec{x}_2}{dt}\right]_0+l_1\ddot\theta_{01}\vec{y}_1-l_1\dot\theta_{01}^2\vec{x}_1\\
\vec{\Gamma}(P,S_3/R_0) &= l_3\ddot\theta_{23}\vec{x}_3+l_3\dot\theta_{23}\left[\frac{d\vec{x}_3}{dt}\right]_0+l_3(\ddot\theta_{12}+\ddot\theta_{01})\sin\theta_{23}\vec{y}_2+l_3(\dot\theta_{12}+\dot\theta_{01})\dot\theta_{23}\cos\theta_{23}\vec{y}_2\\
&- l_3(\dot\theta_{12}+\dot\theta_{01})^2\sin\theta_{23}\vec{x}_2-l_2(\ddot\theta_{12}+\ddot\theta_{01})\vec{x}_2-l_2(\dot\theta_{12}+\dot\theta_{01})^2\vec{y}_2+l_1\ddot\theta_{01}\vec{y}_1-l_1\dot\theta_{01}^2\vec{x}_1\\
&= l_3\ddot\theta_{23}\vec{x}_3+l_3\dot\theta_{23}\left[(\dot\theta_{12}+\dot\theta_{01})\vec{z}_2+\dot\theta_{23}\vec{y}_2\right]\wedge\vec{x}_3\\
&- l_3(\dot\theta_{12}+\dot\theta_{01})^2\sin\theta_{23}\vec{x}_2-l_2(\ddot\theta_{12}+\ddot\theta_{01})\vec{x}_2-l_2(\dot\theta_{12}+\dot\theta_{01})^2\vec{y}_2+l_1\ddot\theta_{01}\vec{y}_1-l_1\dot\theta_{01}^2\vec{x}_1\\
&+ l_3(\ddot\theta_{12}+\ddot\theta_{01})\sin\theta_{23}\vec{y}_2+l_3(\dot\theta_{12}+\dot\theta_{01})\dot\theta_{23}\cos\theta_{23}\vec{y}_2
\end{aligned}$$

$$\vec{\Gamma}(P,S_3/R_0) = l_3\ddot\theta_{23}\vec{x}_3+2l_3\dot\theta_{23}(\dot\theta_{12}+\dot\theta_{01})\cos\theta_{23}\vec{y}_2-l_3\dot\theta_{23}^2\vec{z}_3+l_3(\ddot\theta_{12}+\ddot\theta_{01})\sin\theta_{23}\vec{y}_2$$
$$-l_3(\dot\theta_{12}+\dot\theta_{01})^2\sin\theta_{23}\vec{x}_2-l_2(\ddot\theta_{12}+\ddot\theta_{01})\vec{x}_2-l_2(\dot\theta_{12}+\dot\theta_{01})^2\vec{y}_2+l_1\ddot\theta_{01}\vec{y}_1-l_1\dot\theta_{01}^2\vec{x}_1$$

$$\boxed{\vec{\Gamma}(P,S_3/R_0)=\left|\begin{matrix}
l_3\ddot\theta_{23}\cos\theta_{23}-\left[l_3\dot\theta_{23}^2+l_3(\dot\theta_{12}+\dot\theta_{01})^2\right]\sin\theta_{23}-l_2(\ddot\theta_{12}+\ddot\theta_{01})+l_1\ddot\theta_{01}\sin\theta_{12}-l_1\dot\theta_{01}^2\cos\theta_{12}\\
2l_3\dot\theta_{23}(\dot\theta_{12}+\dot\theta_{01})\cos\theta_{23}-l_2(\dot\theta_{12}+\dot\theta_{01})^2+l_1\ddot\theta_{01}\cos\theta_{12}+l_1\dot\theta_{01}^2\sin\theta_{12}\\
-\left[l_3\ddot\theta_{23}+l_2(\ddot\theta_{12}+\ddot\theta_{01})\right]\sin\theta_{23}-l_3\dot\theta_{23}^2\cos\theta_{23}-l_3(\dot\theta_{12}+\dot\theta_{01})^2\sin^2\theta_{23}
\end{matrix}\right._{B2}}$$

En considérant que $\dot\theta_{12}=\text{constante}$; $\dot\theta_{01}=\text{constante}$; $\dot\theta_{23}=\text{constante}$, on obtient ;

$$\boxed{\vec{\Gamma}(P,S_3/R_0)=\left|\begin{matrix}
-\left[l_3\dot\theta_{23}^2+l_3(\dot\theta_{12}+\dot\theta_{01})^2\right]\sin\theta_{23}-l_1\dot\theta_{01}^2\cos\theta_{12}\\
2l_3\dot\theta_{23}(\dot\theta_{12}+\dot\theta_{01})\cos\theta_{23}-l_2(\dot\theta_{12}+\dot\theta_{01})^2+l_1\dot\theta_{01}^2\sin\theta_{12}\\
-l_3\dot\theta_{23}^2\cos\theta_{23}-l_3(\dot\theta_{12}+\dot\theta_{01})^2\sin^2\theta_{23}
\end{matrix}\right._{B2}}$$

Question 6 :

En considérant que les vitesses articulaires sont données sur la figure 4, déterminer les variables articulaires correspondantes θ_{01}, θ_{12} et θ_{23} en fonction du temps pour $t \in [17s, 27s]$. On prendra $\theta_{01}(t=0)=0, \theta_{12}(t=0)=0$.

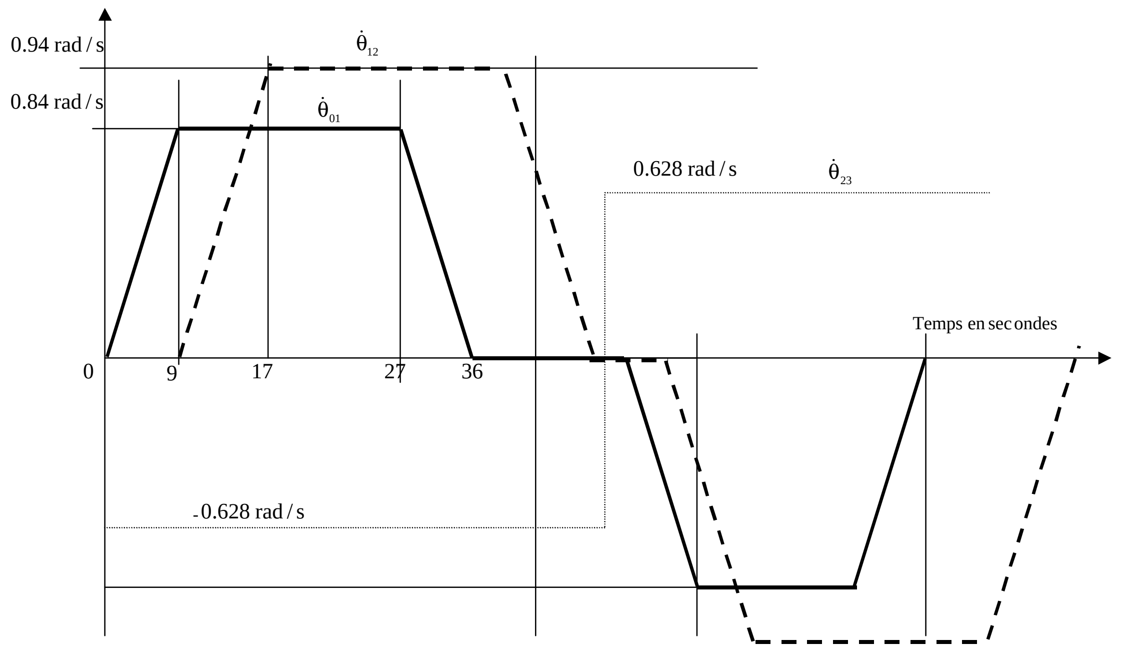

Figure 4 : Vitesses articulaires du manège

$t \in [0, 9]$	$\dot\theta_{01} = 0.093t$ $\dot\theta_{12} = 0$ $\dot\theta_{23} = -0.628$	$\theta_{01} = 0.046t^2$ $\theta_{12} = 0$ $\theta_{23} = -0.628t$	$\theta_{01}(9) = 3.78$ rad $\theta_{12}(9) = 0$ rad $\theta_{23}(9) = 5.65$ rad
$t \in [9, 17s]$	$\dot\theta_{01} = 0.84$ $\dot\theta_{12} = 0.1175t - 1.0575$ $\dot\theta_{23} = -0.628$	$\theta_{01} = 0.84t - 3.78$ $\theta_{12} = 0.05875t^2 - 1.0575t + 4.758$ $\theta_{23} = -0.628t$	$\theta_{01}(17) = 10.5$ rad $\theta_{12}(17) = 3.76$rad $\theta_{23}(17) = -10.676$ rad
$t \in [17s, 27s]$	$\dot\theta_{01} = 0.84$ $\dot\theta_{12} = 0.94$ $\dot\theta_{23} = -0.628$	$\theta_{01} = 0.84t - 3.78$ $\theta_{12} = 0.94(t-17) - 3.76$ $\theta_{23} = -0.628t$	$\theta_{01}(27) = 18.9$ rad $\theta_{12}(27) = 5.64$ rad $\theta_{23}(27) = -16.95$ rad